Acknowledgement of priority concerning "On the central limit theorem for geometrically ergodic Markov chains"

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The statement of [3, Thm. 1.3] is that there exists a geometrically ergodic Markov chain whose stationary distribution has a finite second moment, but for which nevertheless the $\frac{1}{\sqrt{n}}$-normalized partial sums converge neither to a normal distribution, nor to a point mass. However, as Richard Bradley has kindly pointed out to me, an example that turns out to have the required properties appeared already in [1, proof of Thm. 2]; see in particular [1, Rem. 2.1]. For the desired counterexample, one can for instance choose parameter values $d_n = e^{-n}$ and $h_n = n^{-0.01}$, with notation as in [1, Thm. 2]. To see this, note (a) that geometric ergodicity of the Markov chain follows – due to a result in [4] – from the exponential rate absolute regularity established in [1], and (b) that the nonconvergence of the $\frac{1}{\sqrt{n}}$-normalized partial sums to a point mass follows from the so-called complete dissipation property, which holds for this example as remarked in [2, p. 545].

References

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