Complexity for Probabilists

Comments and partial solutions to homework 1

1. (a) and (b) did not seem to cause that many problems.

   As to (c), some of you have misunderstood the class of functions that are to be shown to be primitive recursive. The task is to show that for any $n \in \mathbf{N}$ and any $i_0, i_1, \ldots, i_{n-1} \in \mathbf{N}$, the function

$$f_3(x) = \begin{cases} i_0 & \text{if } x = 0 \\ i_1 & \text{if } x = 1 \\ \vdots & \vdots \\ i_{n-1} & \text{if } x = n-1 \\ 0 & \text{otherwise} \end{cases}$$

   is primitive recursive.

2. Suppose that there exists an algorithm $M$ which on input $x \in \{1, 2, \ldots\}$ outputs the $x$:th smallest element of $A$. Then we can construct an algorithm $M'$ which on input $y$ decides whether or not $y \in A$: just compute $M(1), M(2), \ldots$ until we get an answer $M(k)$ such that $M(k) \geq y$. If $M(k) = y$ then $y \in A$, otherwise not.

   Hence $A$ is recursive. But Theorem 2.14 in Håstad's notes tells us that there exist recursively enumerable sets that are not recursive, and hence the answer is **no**.

3. **Dead code.** The answer is **no** again. Let me simply quote Koen Claessen's slick solution:

   > If such a program existed, one could solve the Halting Problem by adding a dummy line to a program and asking if that line is ever executed. So clearly the answer must be "no".

4. **Goldbach's conjecture.** This is just a trick question! For this reason I'm not keeping any records of whom I managed to "fool" with it.

   The answer is **yes**, for the following reason. $f(x)$ does not depend on $x$ (either Goldbach's conjecture is true, and then $f$ is the constant function 1, or Goldbach's conjecture is false, and then $f$ is the constant function 0). But constant functions are recursive and therefore computable.

   Intuitionists may disagree, but then again, I do not care much for (or perhaps have failed to understand) intuitionism.