Complexity for Probabilists

Comments and partial solutions to homework 2

1. This problem you all solved (with a wide variety of different sorting algorithms!), so no comments are needed.

2. No comments needed for this one either.

3. This one was (as I expected) much trickier. Let us call the problem MWL (Minimal Wire Length). The idea I had in mind was to show that the TSP (Traveling Salesman Problem) can be reduced, in polynomial time, to MWL. Since TSP is NP-complete, we then have that MWL is NP-hard. Hence, assuming that P$\neq$NP, there is no polynomial time algorithm for MWL.

   The reduction is as follows. Suppose we want to solve TSP on n cities with a given $(n \times n)$ distance matrix D. Let $k = n$, let the distance matrix for the locations in MWL problem be the same as for the TSP problem, and let the "wire matrix" A be of the form

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   so that in other words the wires form a tour through all the units. But with this choice of A, the solution to the MWL problem gives the shortest tour through all the locations, and therefore solves the TSP problem.

   Two of you have found a different solution in the same spirit: to show that the so-called CLIQUE problem, which is also known to be NP-complete, can be reduced in polynomial time to the MWL problem. Nice!