COMPLEXITY FOR PROBABILISTS
HOMEWORK 2

1. In the lecture of January 21, I described the sorting algorithm MERGESORT. Suggest (at least) one other sorting algorithm, and analyze its time complexity.

2. **Minimal spanning trees.** The following problem looks, at first sight, similar to the traveling salesman problem, but is in fact much easier (in the sense that there are much faster known algorithms for this problem). We are given n cities and an $n \times n$ matrix containing the distances between the cities. We want to build a railroad system that connects all the cities, while minimizing the total length of the railroads. This is done by a suitable choice of pairs of cities to build a railroad between. Suggest an algorithm[1] that sloves this problem, and analyze its time complexity.

3. Here's a problem that might be of some relevance in industrial design. We are given k units (say, processors) that will form some electronic device (say, a computer), and a $k \times k$ matrix indicating the number of wires needed between each pair of units. We are also given a set of n locations (with $n \geq k$) in the plastic hull of the computer, and an $n \times n$ matrix containing the distances between these locations. We want to find the placements of the k units on the n possible locations (with at most one unit at each location) that minimizes the total length of the wires. Does there exist a polynomial (in k and n) time algorithm for solving this problem?[2]

[1] If you get stuck in trying to think of such an algorithm, you can search for "Prim's algorithm" or "Kruskal's algorithm" in the literature.

[2] I expect that most of you will not arrive at a definite "yes" or "no" answer to this question. But I do hope that you will be able to make an educated and well-motivated guess, perhaps based on the famous "$P \neq NP$" conjecture.