

LARGE DEVIATIONS
HOMEWORK 6

Deadline for handing in solutions: April 30.

1. Consider a Markov chain (X_1, X_2, \dots) with state space $\{0, 1\}$. In particular, consider the two cases where transition matrix is given by either

$$P = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

or

$$P' = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}.$$

In both cases, the average $M_n = \frac{1}{n}(X_1 + \dots + X_n)$ tends to $\frac{1}{2}$ with probability 1 (right?).

- (a) Compute, in both cases (P and P'), the exponential decay rate of the probability of the large deviation event $\{M_n > \frac{3}{4}\}$.
 - (b) In both cases, $\sqrt{n}(M_n - \frac{1}{2})$ satisfies a central limit theorem. What is the variance of the limiting normal distribution in the two cases? Compare the answer to that in (a).
2. Suggest a generalization of den Hollander's Theorem II.18 to a situation where the X_i 's are no longer i.i.d., but instead a Markov chain.