NUMERICAL SIMULATION OF LASER TRANSMISSION AND SOLITON

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Laser transmission is described by nonlinear Schrodinger system. When dispersion $K > 0$, by use of a relative time $\tau = c(t - z/v)$, it becomes a canonical form

$$iu_z + \frac{1}{2}(u_{xx} + u_{yy} + u_{\tau\tau}) + |u|^2 u = 0. \tag{1}$$

There are five kinds of difficulty for its numerical solution: 1). High dimension; 2). Nonlinearity; 3). Unbounded domain; 4). Long time; 5). How choose a better initial value, if search its soliton. The computation of this problem is so compicated that no way to begin. Follow Y.Silberberg, we discuss the cases of sphere ($m = 2$) and cylinder ($m = 1$) symmetry, which are decreased to $1 + 1D$, but with a singular coefficient,

$$iu_z = \frac{1}{2}(u_{rr} + \frac{1}{r}u_r) + |u|^2 u = 0, u0, r = \phi(r), \quad u(z,0) = 1,\ u(z,\infty) = 0,\ z, r > 0. \tag{2}$$

By a model analysis, $u = U(r)exp(i\lambda z)$, it leads to a nonlinear eigenvalue problem

$$\frac{1}{2}(U_{rr} + \frac{m}{r}U_r) - \lambda U + U^3 = 0,\ U(r) > 0, U(0) = 1, U(\infty) = 0. \tag{3}$$

He has not explained how to solve it numerically, only given a few data and a sketch. In this paper, we shall propose an effective algorithm to calculate its soliton solution and then further study their properties. First, to solve (3), we adopt the following treatments:

1). Turn to discuss a corresponding initial value problem with $U(0) = 1, U'(0) = 0$ and different λ;

2). Use a continuous finite element method with weights r in $(0, 1)$ and r^m in $(1, \infty)$, so that the conservation for large r is retained.

3). Use the interpolated coifficient finite element method to treat the nonlinear term;

4). Adopt a new algorithm to choose the parameter λ, so that a desired solution $U(r, \lambda) \geq 0, U(R, \lambda) \approx 0$ in larger interval $(0, R)$ can be found.

Our numerical computations show that the desired eigenvalue λ and corresponding solution $U(r)$ are obtained and it is found that the solution $U(r)$ decreases exponentially in larger interval $(2, 20)$, then $U' + bU = 0$, where the positive constant b is determined. In these computations, we found that the behavior of solution is very sensitive to the change of λ, and a lot of interesting phenomenon is observed.

Finally, we take fully discrete finite element scheme to solve (2) with the inital value $\phi = U(r)$ and artificial boundary values $u(z, 0) = 1, u_r + bu = 0$ on $r = R$, where we can choose R a not large number. A long time computation show that $u(z, r) = U(r)exp(i\lambda z)$, and then $|u(s, r)| = U(r)$ is ineed a desired soliton solution.