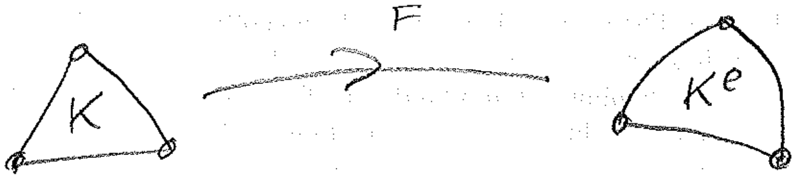
Lecture 13

4.7 Isoparametric elements



ϕ_N

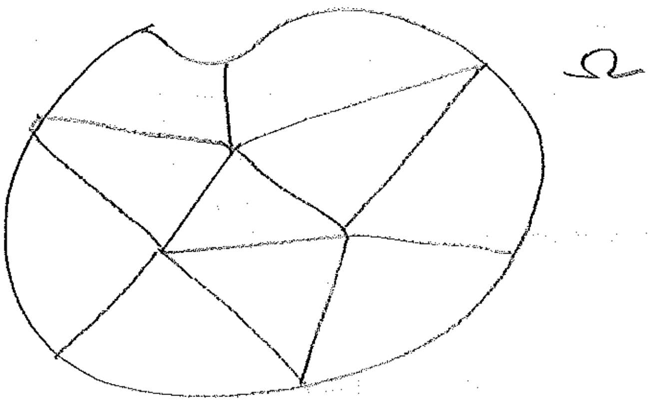
$(K, \mathcal{P}, \mathcal{N})$

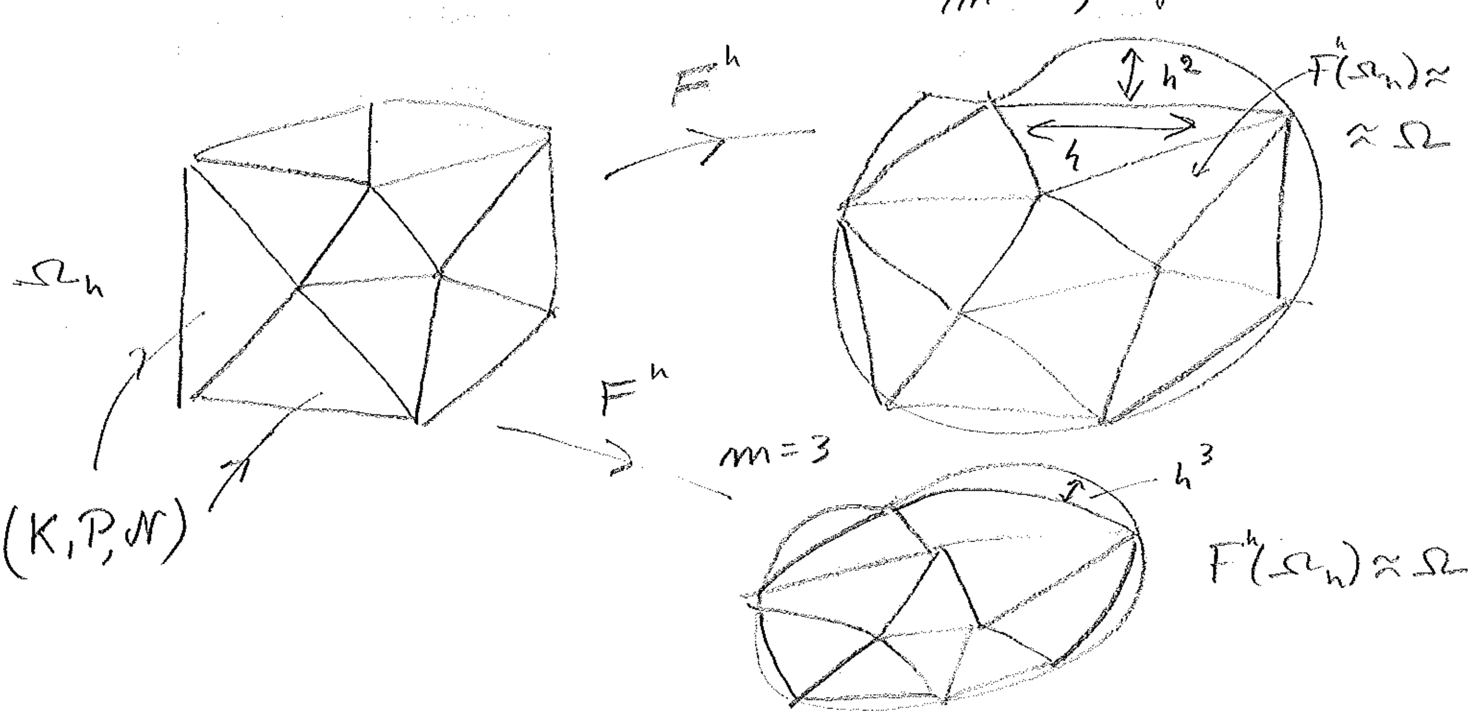
$\phi_N^e = (F^{-1})^*(\phi_N)$

$\phi_N^e(x) = \phi_N(F^{-1}(x))$

affine: $F(x) = ax + b$

isoparametric: $F \in \mathcal{P}$





Ω smooth domain $\subset \mathbb{R}^n$

Ω_h polygonal domain

T^h subdivision of Ω_h

F^h piecewise $\in \mathcal{P}_{m-1}$ such that

1. $F^h = I$ away from $\partial\Omega_h$
2. $\mathrm{dist}(\partial\Omega, \partial\Omega_h) \le C h^m$
3. $$\left.\begin{aligned} \| J_{F^h} \|_{W^m_\infty(\Omega_h)} &\le C \\ \| J^{-1}_{F^h} \|_{W^m_\infty(\Omega_h)} &\le C \end{aligned}\right\} \text{ shape!}$$

4.7.3 Thm

$\{T^h\}$ non-deg. of $\Omega_h \approx \Omega$

F^h as above

$(K, \mathcal{P}, \mathcal{N})$ C^0 ref. element $\left(m - \frac{n}{p} > 0 \right)$

$(T, \mathcal{P}_T, \mathcal{N}_T) \hat{=} (K, \mathcal{P}, \mathcal{N})$

Then, $k = 0, 1$

$$\| v - I^h v \|_{W^k_p(F^h(\Omega_h))} \le C h^{m-k} |v|_{W^m_p(F^h(\Omega_h))}$$

$$(I^h v)(F^h(x)) = (\tilde{I}^h \tilde{v})(x), \quad \tilde{v} = (F^h)^*(v)$$

$$\tilde{v}(x) = v(F^h(x)), \quad x \in \Omega_h$$

$\tilde{I}^h$ global interpolator on Ω_h