Lecture 2 Jan 20, 2012

Chapter 0 1-D $d=1,\ \Omega = I = (0,1)$

$$\begin{cases} -(a u')' = f \text{ in } I \\ u(0) = u(1) = 0 \end{cases} \qquad 0 < a_0 \le a(x) \le \|a\|_{L_\infty}$$

$$\begin{cases} u \in H_0^1 = H_0^1(I) \\ (a u', v') = (f, v) \quad \forall v \in H_0^1 \end{cases}$$

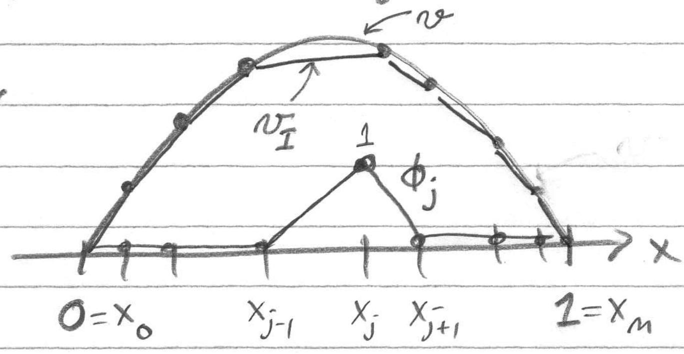
$$\|v\|_E = \left(\int_I a |u'|^2 dx \right)^{1/2} = \|\sqrt{a}\, u'\|$$

$$|v|_{H^1} = \|v'\|, \quad |v|_{H^2} = \|v''\|.$$

Unique solution: $u \in H_0^1$

Regularity: $u \in H^2, \quad \|u''\| \le C \|f\|$

Family of meshes.



Notation:

$I_j = (x_{j-1}, x_j)$

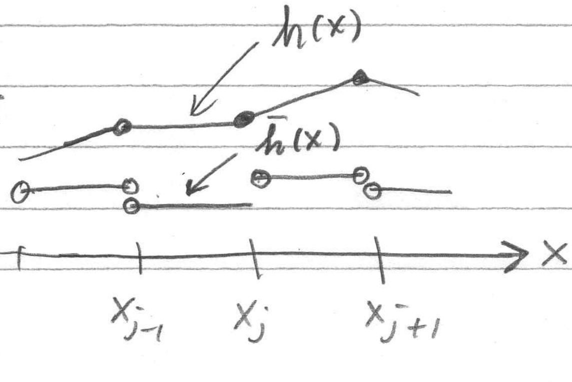
$h_j = x_j - x_{j-1}$

$h_{max} = \max h_j$

$$S = \{ v \in C[0,1] : v|_{I_j} \in \Pi_1,\ v(0) = v(1) = 0 \}$$

FEM

$$\begin{cases} u_S \in S \\ (a u_S', \chi) = (f, \chi) \quad \forall \chi \in S \end{cases}$$



Continue with notes on "Weighted estimates".