

## Let Platonism Live!

Why does Platonism excite such a widespread mixture of aversion and embarrassment? In his essay titled 'Let Platonism Die' published in the June issue of the EMS Newsletter, as well in a similar essay published in the Notices of the AMS some year ago, Brian Davies launches an attack on Platonism in mathematics. He is, as noted far from being alone. Reuben Hersh has during recent decades in a series of books, starting with the 'Experience of Mathematics,' both presented an eloquent case for mathematics being a humanistic subject on par with law, religion and art say, as well as disowning and disparaging Platonism, in fact linking Platonist sympathies with rightest political views.

Issues of philosophy, especially in their metaphysical aspects, tend to make many mathematicians uncomfortable, but any meaningful discussion of Platonism cannot avoid it. Metaphysics is really on the fringe of what can be expressed and rationally argued upon, hence notoriously resistant to interesting discourse, yet as noted inescapable, because as the British philosopher Collingwood reminds us, to reject metaphysics, is by itself a metaphysical stand. Philosophers like Bertrand Russell and Karl Popper tend to think of metaphysics as proto-theories, i.e. as undeveloped thought, and once developed as ceasing to be metaphysical. While metaphysics as developed thought, most notably in the mind of Hegel, gave to the subject by the end of the 19th century a very bad reputation<sup>1</sup> and an almost universal ambition among philosophers to relieve philosophy of superstitious ballast, an ambition still prevalent, if no longer as strident.

Now the philosophy of mathematics has after the pragmatic closure of the crisis of its foundations about a century ago, been largely left to the philosophers. However, I agree with Hersh, that mathematicians should reclaim the territory, after all they make up the indigenous population. In my view Hersh is mostly interested in the practice of mathematics, on which he has many incisive things to say, but differs from me, and many (most?) mathematicians in denying its metaphysical basis, and thereby, as noted above, making a major metaphysical claim.

First what is Platonism? As mathematicians we are used to precise definitions, otherwise we surely cannot properly discuss matters, but will flounder into a bog of confusion and ultimate nonsense. However, we are on the other hand in everyday contexts quite comfortable with discussing concepts for which we are unable to provide exact definitions. 'Intelligence' is one obvious such example<sup>2</sup>. We cannot give a precise definition, yet most of us would be offended if it was indicated that we lacked intelligence, although by virtue of our putative intelligence we would prefer not to acknowledge such offenses. Thus in the spirit of the present discussion I will not try to make Platonism precise, instead I will try to evoke what it might be, and what it definitely is not.

First of all Plato is a historical figure and there is a large oeuvre of writing attributed to him, mostly in the form of so called dialogues, i.e. dramatizations of intellectual inquiry. This oeuvre forms the basis of literally centuries of analysis and commentary, which makes up a body of work and an academic sub-discipline

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<sup>1</sup>The reader who would like to savour sarcasm directed against Hegel, may find in the writings of the pragmatic philosopher and renowned psychologist William James much of what is quotable.

<sup>2</sup>The old quip that intelligence is what is measured by intelligence tests, is of course deeply unsatisfying, IQ is what is measured by intelligence tests, and what IQ has to do with 'intelligence' is an open question.

the details of which it would be both impractical and pretentious to delve into, and whose overriding ambition it is to find out what Plato really meant. This is a large ambition, even if it only pertains to a single intellect, and is bound never to be completed, because after all, as in all historical inquiry, the documentary evidence is fragmentary, and in the case of Plato further complicated by the fact that he tends to be very ironic. What emerges out of those studies is not entirely a very sympathetic picture. Popper, notably, in his monumental 'The Open Society and its Enemies' suggests the spectre of an authoritarian fascist at heart, far from the prevailing political sensibilities of our days. And even if we try to ignore the man and concentrate on his ideas, the notion of a Platonic realm, timeless and abstract, beyond the physical world of the senses, seems couched in an outdated religious language. And in fact Platonism, or more precisely Neo-Platonism, as developed by among others Aristotle did play an important role in furnishing emerging Christianity with its intellectual credentials. In addition many of the things that Plato claims, such as universal forms, seem naive in their rigidity, not to say outright silly, when literally applied to particular examples. And would we look into his detailed elaborations of how the universe looks like, it is hard to detect anything of interest beyond that pertaining to a certain historical curiosity. (Although once again in view of Plato's penchant for irony, I would caution any reader to take him too literally.) In fact has not his philosophy been superseded by its modern development, just as classical science and medicine has been made obsolete by the tremendous advantages of recent times? Platonism seems only to survive in fossilized form among philosophically unsophisticated mathematicians as well as among overly enthusiastic cosmologists. In the words of Brian Davies, echoing the war-cries of the trend-setting philosophers of a hundred years ago, it is time to throw out this ballast of ancient superstition. However, I do not think that one can dismiss Platonism so easily, and furthermore I fail to see any merits in the specific arguments Davies musters in his article<sup>3</sup>, nor any suggestions of illuminating prospects he claims that the rejection of Platonism opens up. I will return to those issues below.

In order to appreciate Platonism as a purely intellectual edifice, we will have to ignore its particular historical manifestation, and in particular ignore speculations of what the individual Plato may have thought. In fact dismiss as irrelevant the fact that there has ever been a historical individual Plato at all<sup>4</sup>. The basis of Platonism is that the world of the senses is a confusing and misleading world, and just a more or less arbitrary manifestation of a deeper, more abstract world. Now most people have some trouble taking this literally, so let me for the moment just explore the metaphorical aspects of it, leaving aside for later the metaphysical injunctions.

The world of natural phenomena is indeed a very rich, complicated and inevitably confusing world, and scientific explanations, often referred to as theories, replace the world as we directly encounter it, with simple principles using concepts that seemingly have nothing to do with the phenomena they are called upon to explain. In what sense those theories are true, or just arbitrary and interchangeable, if useful,

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<sup>3</sup>In particular his first example pertaining to visual illusion seems to me singularly irrelevant, if anything it provides an illustration of the confusion of sensual impressions, so central to Platonism

<sup>4</sup>The analogies with Christianity are of course too obvious to ignore. Can there be Christianity without Christ? Will Christianity be rendered moot if Jesus never existed as a historical person? Most Christians would, I suspect, be made very uncomfortable with the suggestion of his non-existence. I claim of course the obvious, that there is Platonism without Plato, any other claim would obviously be absurd.

figments of our imaginations is a fundamental philosophical question, but it is an undeniable fact that the creation of explanatory theories constitute the ambition of all science, and that the (provisional) acceptance of those theories are based on a combination of aesthetic approval, i.e. appeals to our reason and empirical verifications (or rather by applying Poppers falsifiability criterion, being in concordance with all other facts and circumstances so far known). In mathematics the phenomenon is all pervasive. Isolated facts being explained by a hierarchy of unifying principles, the discoveries of which provide one of the basic intellectual drives of doing mathematics. Furthermore mathematics abound in isomorphic concepts, which thus should be considered the same, although it is impossible to formulate such concepts in an all-inclusive way, without just adding yet another instance of the same. The notion of number is of course an obvious example. Thus the 'true' concept 'exists' beyond that of any of its countless manifestations. A less technical, and hence less amenable to articulation, is the abstract concept of a mathematical idea. This is something that cannot be formulated say as a theorem, only having some of its aspects manifested as such, but constituting an inescapable prerequisite for all mathematical activity. Here we are touching upon the distinction between formalism and meaning, of which the former is but a manifestation of the latter. Raised to a metaphysical level this ties in with the dualism between matter and mind, another source of embarrassment to modern science.

The reader may argue that the presentation of Platonism so far is too vague and inspid and as such unobjectionable and so generally applicable as to have no real content at all. In order to make it have more precise contours we need to contrast it to alternatives. According to Hersh mathematics is but a human invention on par with law, language, and art say (into which we may also include religion), each of which would make no sense without humans. Human invention is close to social convention, and clearly law, language and art transcend individual subjectivity and their apparent objective features only emerge in a social and historical context. One illustrative example is money, if ever a social convention. The denominations on pieces of papers suggest no intrinsic value, but are conventions, yet conventions which the individual is frustratingly unable to flout on his own, being as compelled by them as he is by gravity. Similar for languages, which seem to obey their own intrinsic laws, and whose changes and developments seem utterly beyond the control of the single individual<sup>5</sup>. As noted, the examples can be multiplied. So are the constraints the individual mathematician experiences in his explorations nothing but social conventions? The palpable reality of the subject matter but an illusion? Is it like the stars and galaxies studied by astronomers, nothing but dark spots of silver on transparent sheets held in the hand? What strikes the practising mathematician is the way that everything in mathematics fit together, how the results of his thoughts evade his wishes, and how facts, just as in real life, are liable to kick back at him. To dismiss this as mere opinion, as Davies suggests, is deeply inconsistent; because it is the same process of verification by hypothesis and experiment at work in mathematics as in the so called inductive sciences<sup>6</sup> It is undeniably true that mathematics is practised by mathematicians,

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<sup>5</sup>Of course individual words can sometimes be traced to specific individuals, but the individual can only suggest never legislate, and the forces of acceptance are mysterious. When it comes to deeper structures of language, the individual has no say whatsoever.

<sup>6</sup>It is an important fact, and here I am in agreement with Hersh, that conviction in mathematics is not based on deduction alone, but on how new facts fit with already known ones. And no

that not only the concepts developed, the notations and nomenclature induced are but human inventions, but also the choice of what to study and thus the definition of what is mathematics, is but a social and historical accident. But I claim that there is a distinction between mathematics and the practice of mathematics, and while the latter is but a human activity, the former is independent of humans. Or to put it slightly differently. That the human activity of doing mathematics is centered around an inhuman core. That one can very well be quite emotional about mathematics, but mathematics provides, unlike art and religion, no vehicle for the expressions of emotions. One may argue that the distinction is a chimera, while it is (obviously) impossible to formulate what is mathematics in ways which are independent of human thought. On the other hand most people of a realist bent have no problem with making a distinction between physics as a human activity and the physical world, which endows the former with its ultimate justification. In the same way Platonism claims that there is a mathematical world, as real and palpable, if more elusive than the material. The alternative provided by Hersh would be to posit some kind of Jungian collective unconsciousness as the basis for mathematics<sup>7</sup>. We are now by making those claims stepping over from the mere metaphorical to the metaphysical.

In making a distinction between a physical and mathematical Platonic reality the mathematician exposes himself to embarrassing questions. In the words of the logician Bencarraff, in what way can there be any communication between those two parallel worlds? An embarrassment quite similar to the Cartesian mind-matter duality, which however appropriate in practice, presents on the meta-physical level an affront against our aesthetics of reasoning, in which the so called Occams razor provides an important component. The defence of the mathematician may be reduced to a plaintive assertion of fact, that this is the way the world is, and that it is way beyond his ability as well as responsibility to come up with an explanation. And furthermore, if of a sarcastic bent, he may riposte that anyone who finds this attitude unsatisfactory proves himself thereby of being imbued with a true Platonic ambition. He may also simply claim, that the temporal spatial world is but a shadow of the (mathematical) Platonic world, and if so, the question is simply moot. And true, the ultimate explanations of the physical world, following a tradition starting with Pythagoras, have increasingly been couched in mathematical terms. Thus not only is mathematics independent of us humans, but also independent of the physical universe. In particular, according to cosmologists, 'equations' somehow exist, even if there are nothing to write them onto, and by their existence forcing the material world to come about.

Now to bring back the discussion from a meta-physical level to a scientific one. The discovery of extra-terrestrial intelligence surely would be able to once and for all settle the question of whether mathematics is just an activity constrained to humans. Many people seem to take the possibility as well as desirability of finding such intelligence seriously enough to suggest directing considerable resources to such a search. Personally I find such ambitions misdirected. The (known) universe is just too small, and even if infinite the velocity of light is too slow to make such searches

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mathematician is so consistently logical that he is not charmed by numerical verifications of special examples.

<sup>7</sup>The choice of the affiliation Jungian is of course provocative, yet when it comes to language, the notion seems quite apposite

feasible<sup>8</sup>. Still as a thought-experiment it is intriguing. First and foremost it would pre-suppose intelligence itself being an extra-human feature<sup>9</sup>. Then, granted this, it would be an interesting challenge to identify the mathematical contents of its manifestation, and compare them to ours.

Extra-terrestrial life being no realistic option of study we are reduced to look at the evolution of life taking place under our noses here on earth. Ultimately all our thoughts and ideas are excretions, so to speak, of our brains, including the one I have just written down, and thus ultimately the fruits of evolution. Frege was aghast at the idea of basing the concept of number on something as flimsy as mere psychology, and this is indeed what Davies seems to do in his (second) example. What he refers to appears to be the findings that when it comes to gauging number without the stratagem of counting, people are unable to go beyond three or four, and that their innate abilities in this regard do not differ from those of rats and crows. This is indeed an interesting fact, but it sheds little light on mans mathematical abilities in general nor the notion of number in particular. Ultimately one suspects Davies to suggest an evolutionary and neurobiological explanation for mathematics. It is a common mistake to believe that what is brought about by evolution can also be explained by evolution as simply adaptive solutions. One example of such deluded thinking is the phenomenon of so called evolutionary psychology, in which people with straight faces do what Kipling did tongue in cheek, namely creating 'just-so stories'. Darwins theory of evolution is a beautiful suggestion of how order can emerge out of chaos without any *a priory* design. Albeit corroborated by innumerable findings, its most direct and hence strongest influence on our acceptance is its immediate appeal to our reason by virtue of its simplicity, and as such the Darwinian explanation is very much akin to that of a mathematical insight<sup>10</sup>. The point of Darwins theory is in the last analysis not whether it is True or not (it is after all not a question of religion), but that it provides biological sciences with a coherent narrative source of asking fruitful questions. On a more philosophical level what evolution does is to provide new constellations and thus new possibilities, which in no way can be explained by the route that was taken to achieve them. True, stability of solutions can most naturally be more or less tautologically explained by adaptive pressures, but evolution is creative in the sense of its products occasionally transcending itself. There is no mysticism about this. The human brain is a case in point. Its search for truth and scientific explanations, can of course not be explained by adaptive pressures. To play the card of Cantors diagonal trick, what is the evolutionary advantage of being able to formulate Darwins theory? The human brain is what it is, and by virtue of its more or less accidentally arrived structure it can pose questions that transcend its adapted features<sup>11</sup>. In addition to the human

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<sup>8</sup>And even if feasible, would they necessarily be advisable?

<sup>9</sup>And how would we be able to recognise this 'intelligence', hardly by giving a standard intelligence test.

<sup>10</sup>It is indeed sometimes claimed that evolution is mathematical in the sense of providing an algorithm. This suggests the notion of evolutionary determinism coupled with genetic determinism, but space and the limited objective of this essay does not permit me to comment on this.

<sup>11</sup>Dawkins, often considered as the genetic reductionist *par excellence* concludes his 'The Selfish Gene' with the admonishment to mankind to break out of the tyranny of genetic survival. Out of which was this transcendent insight derived? Another adaption? In the case of Dawkins, he identifies a deeper selective phenomenon, of which Darwins biological one, is but a manifestation. Once again the limited format of this essay prevents me from pursuing this further.

brain<sup>12</sup>, evolution has likewise in its long history produced multi-cellular organisms and invented sex, thereby changing its course irrevocably, creating, accidentally or not, new facts on the ground. One may think of this as a very Platonic feature as navigation in a huge configuration space of constellations. In a sense it is as if all those constellations do exist outside time, and the history of the universe, or at least its evolutionary one, confined (so far) on our own planet is but an exploration of this huge space. Simply one out of countless other explorations.<sup>13</sup>

One may now be curious as to what contemporary professional mathematical philosophers have to say about Platonism. As a mathematician one is disappointed. Platonism is of course divorced from its accidental historical provenance, and becomes a question of what is meant by existence. Platonism is then contrasted against Nominalism, the latter famously rejects the notion of abstract entities. Thus if we have a collection of  $n$  objects, the nominalists only recognise the existence of  $2^n$  entities altogether, while the Platonist applies the process inductively and creates an infinity of entities, such as sets of sets of sets of subsets etc. More seriously, in the view of the logicians, any collection of objects subjected to consistent axioms are supposed to exist. The idea of reducing mathematics to the frivolous play of arbitrary axioms is abhorrent to most of us. In a way it is a materialistic view of mathematics, in which everything is reduced to atoms subjected to some general laws of composition. It is subtracting from mathematics all vestiges of meaning. Of course by so doing, concrete objects of study are created, namely the formal systems themselves. But those systems are not as interesting as those some of them are supposed to describe. (Furthermore by numerical encoding we reduce them to, albeit artificial, problems in numbertheory, going back to square one) Needless to say the question of whether any interesting system of axioms is internally consistent is far from a trivial question, in fact one which in principle cannot be settled. Thus mathematicians take things on faith, and their work can be viewed as a continual testing of hypothesis in the sense of Popper. In particular persuasion to mathematicians do not mainly come from long deductive chains of reasoning, but from the way everything fits with everything else, which cannot be stressed too often.

Ultimately Platonism is about Truth with a capital T. The assumption that an absolute Truth exists, even as we as feeble human beings only can hope to approximate it<sup>14</sup>. As such Platonism at its inception took a definite stand against the Sophists, to whom Truth was just a social convention. The Sophists were deplored by Socrates, as he appears in one of Platos dialogues<sup>15</sup> to be as eager to argue one side of a question as the other, being more concerned with the appearance

<sup>12</sup>Which the late Maynard-Smith has found convenient to identify with language

<sup>13</sup>This metaphor of a configuration space is of course borrowed from physics and mathematics, and enthusiastically used in popularization of evolutionary theories as a pedagogical device. One may argue whether it is just a cheap mathematical trick of getting rid of time with no deeper metaphysical relevance. A mathematician confronted with something that depends on time (or  $t$ ) simply either integrates over the time variable, or just (what basically amounts to the same thing) adds it as an extra variable. The function  $f(x, t)$  as a function of (two) variables is in fact independent of  $t$  in contradistinction to its values. Similarly Einsteins theory of general relativity taken as a whole, much to the consternation of Popper, is a timeless space-time continuum.

<sup>14</sup>And this is what Popperism is all about. The successive approximation asymptotically approaching Truth, driving our supporting poles deeper and deeper into the marsh, to use one of his many metaphors.

<sup>15</sup>Is Socrates a historical figure, or only a fictional figment of Platos imagination? It really does not matter, which incidentally touches upon another question, namely that of 'fictionalism' as an alternative take on Platonism, for which there will be, once again, no space to delve into in

of cleverness, than with the issues themselves. The Sophists of today are known as the Post-Modernists, and their approach to inquiry is modelled upon social intercourse, where truth is expendable as opposed to persuasion itself, and does not have any intrinsic value, but is interesting only as a mean to an end. By arguing that mathematics is but a social human invention, one does ultimately place oneself into the camp of the Post-Modernists.

Finally why does it matter? Is Platonism True? Maybe the proper way to argue is the pragmatic, namely what difference does it make? If a Platonistic attitude is useful and makes sense, as well as being fortifying and inspiring, it certainly should be pursued and upheld, just as I argued above about Darwinism. This attitude itself may be dangerously close to that of Post-Modernists, but this is just part of the irony in involving in social debate about existential issues.